A Virtual School Mathematics Laboratory

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A Virtual School Mathematics Laboratory: The paper deals with a *Virtual School Mathematics Laboratory (VirMathLab)* being developed at the *Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences (IMI-BAS).* Various ways of using it in mathematics education are presented, with emphasis on the implementation of the Inquiry Based Learning (IBL) by means of dynamic geometry software. Strategies for integrating it with the education of pre-service teachers in mathematics and information technologies are discussed.

Keywords: *Inquiry Based Mathematics Education, dynamic resources, mathematics laboratory*

INTRODUCTION

Although the computers have been widely spread in the Bulgarian schools (even from the early 80s) there is still a significant number of teachers who do not feel comfortable with using computers in their mathematics classes. Even if they do, this is mainly to introduce information from the Internet and illustrate an idea with static pictures in *Power Point*. But most of what computers can offer in mathematics education has nothing to do with the information in the ordinary sense – the information (or better, *digital)* technology can provide learning environments which could bring the real spirit of mathematics as a science. In such environments students can carry out experiments, formulate hypotheses, get ideas how to prove or reject them, and the teacher would act as a guide/partner in a research team. If our policy makers smile skeptically: "our teachers are not ready for this", our reaction would be: "Yes, if the teachers are not given the chance to learn how". And such a chance should be given to them early enough – in their pre-service education. It is at the university level that the future teachers should learn how to use the computer as a laboratory for explorations in mathematics and how to encourage his/her students to learn in the same inquiry based spirit.

THE INQUIRY BASED MATHEMATICS EDUCATION IN A BULGARIAN SETTING

The appearance of software environments of laboratory type in the recent decades ranging from Logo to GeoGebra [6], [2] enhances the implementation of the inquiry based mathematics education, i.e. education in which mathematics is studied similarly to the natural sciences – by making experiments and observations, looking for patterns, formulating and verifying conjectures and hypotheses.

Currently there is free access to such systems, the communication could be tuned to the national language and it is advisable that they could be spread not only in the context of mathematics education but also when studying physics, chemistry, and biology in a natural integration with mathematics.

Today the inquiry based learning is regaining its popularity thanks to a number of European projects – the most recent include *InnoMathEd, Fibonacci, Meetings in Mathematics, Math2Earth, DynaMat, Mascil, KeyCoMath, Scientix2* (in which IMI-BAS is the Bulgarian partner).

In the frames of these projects various learning environments (dynamic scenarios with a relevant methodological support) providing an appropriate platform for IBL have been developed and tested in a class setting and in out-of class activities. Hundreds of teachers in mathematics and information technologies have been included in seminars and workshops in the frames of the above projects and then, the most active of the teachers have participated in the approbation, refinement and enrichment of these environments so as to tune them to the needs of their pupils.

Since the inquiry-based learning in science education is described as occurring at four main levels [1], viz. (i) *confirmation inquiry*; (ii) *structured inquiry*; (iii) *guided inquiry*, and (iv) *open inquiry*, it is important to note that during the project seminars and professional development courses for in-service and pre-service teachers we have prepared the participants to work at all these levels and possible mixture of those (*coupled inquiry*). This means that the teachers have been expected to learn:

- to use the dynamic scenarios so that the learners confirm a principle through an activity when the results are known and confirmation of the results is the object of the inquiry (*confirmation inquiry*);

- to modify the dynamic scenarios so that the learners investigate a teacher-presented question through a prescribed procedure (e.g. exploring a dynamic geometry construction), while generating an explanation supported by the evidence gathered through the investigation (*structured inquiry*);

- to create their own dynamic scenarios so that the learners investigate a teacher-presented question using procedures designed (or selected) by themselves (*guided inquiry*).

As expected some students started using the dynamic resources out of class, creating their own dynamic constructions so as to investigate questions formulated by themselves (*open inquiry*)**.**

Thus, with the intensive use of digital technologies in a school setting we have witnessed some inspirational practices of high school students studying mathematics in the style of "working mathematicians", i.e. using scientific methods as an integrated whole; conducting their own explorations; formulating hypotheses and problems, and attacking open problems [4].

It is clear that the digital technologies have a strong impact on the way mathematics is taught and learned, on the way one access the mathematics knowledge and the mathematics *instruments* built on its basis.

CREATING A VIRTUAL SCHOOL MATHEMATICS LABORATORY AT IMI-BAS

The idea of preparing and developing a repository of e-resources which could be used in support of inquiry based mathematics education at different levels and forms led to the establishment of a *Virtual School Mathematics Laboratory (VirMathLab)* at IMI-BAS (http://www.math.bas.bg/omi/cabinet/). (A screen shot of its site is given in Fig.1 illustrating the current distribution of more than 700 dynamic resources by topics and age).

The prevailing part of the dynamic scenarios is developed by using *GeoGebra.* The dynamic files could be used on-line or to be downloaded. The search (envisaged to be performed in several languages) is done by keywords or by part of a word.

The infrastructure reflects the specifics of the organization of mathematics education in Bulgarian schools, e.g. placing the resources for the primary school in a separate section, the geometry resources – in a section for the secondary school, etc.

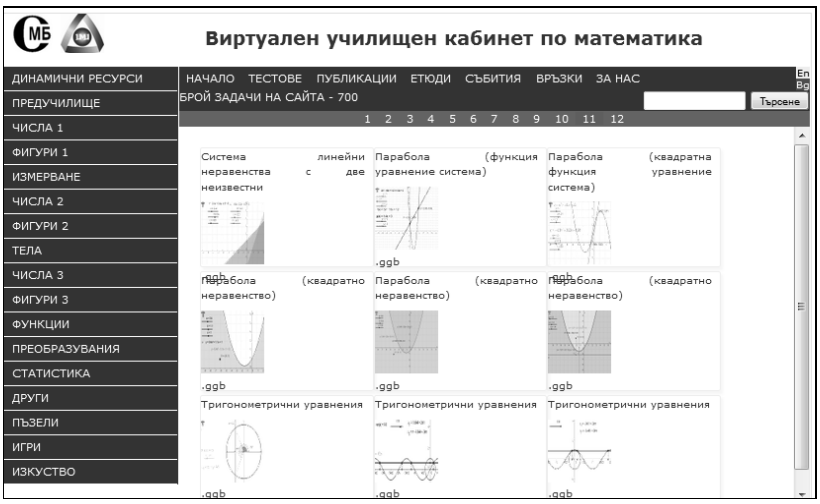

Fig.1. The site of the VirMathLab at IMI-BAS – examples of the "Function" section

Every dynamic file could be considered as a *half-baked* e-resource to be used as a means for: providing conditions for explorations, visualization of the solutions, testing and self-testing, creating and formulating mathematics problems, preparation of didactic resources on paper, solving practical problems with a specific precision, motivation for mathematical or programming activities, acquiring skills for working with a specific software, forming of competencies for working with a text (mathematical and CS alike), development of algorithmic thinking, etc. The users of *VirMathLab* (pupils, teachers, parents, students, researchers) can treat the dynamic files as a basis of reconstruction leading to a model appropriate for accomplishing a goal they are pursuing. For instance, the dynamic file containing a construction of a parabola and a straight line (http://www.math.bas.bg/omi/cabinet/content/bg/html/d19230.html, Fig.2 left) can be used for working with linear functions, quadratic functions, equations, inequalities, systems of a linear and a quadratic equation.

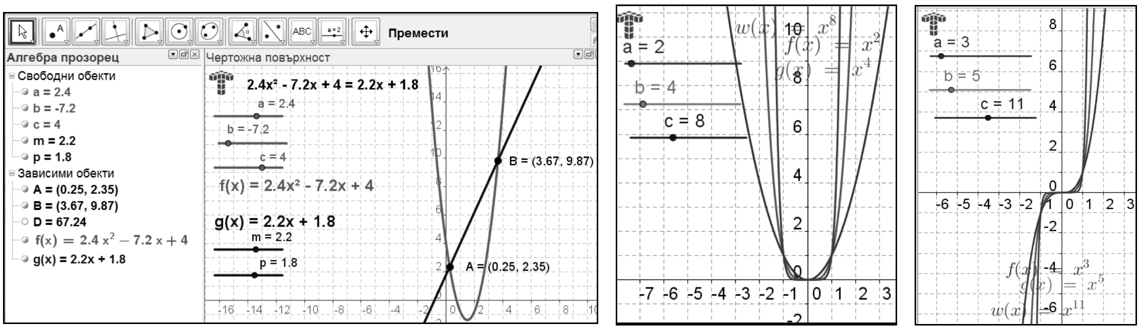

Fig.2. Using an e-resource as a didactical means

To edit this file is easy, including for solving similar problems. The quadratic equation could be replaced by a linear, biquadratic,…, by *n*-th degree one. While varying the parameters and observing the graphs the students can grasp important properties of the functions leading to generalizations, to studying particular cases, to creating new files by analogy (Fig. 2, the right two).

A natural continuation of the investigation is varying the domain of the parameters by including negative values or modifying the step by adding fractions. The teacher might suggest observing specific couples of graphs and looking for relationships which would reveal new properties of the function being investigated. Knowing the full potential of the software as well as its limitations is another important competence for formulating hypotheses. The key competencies in this context include of course the knowledge and skills necessary for evaluating the results obtained by means of computer simulations.

The potential of GeoGebra to enhance the connection between the algebraic and graphical representation of a geometric object enables the students to extract the specifics of an object when exploring different ways of constructing it, e.g. the parabola constructed as a locus of points (http://www.math.bas.bg/omi/cabinet/content/bg/html/d19039.html) or as an envelope of lines (http://www.math.bas.bg/omi/cabinet/content/bg/html/d19040.html) (Fig. 3).

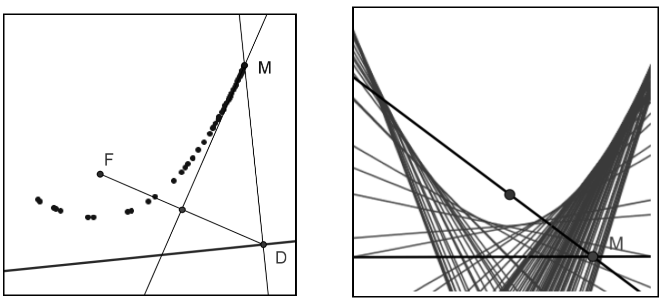

Fig.3. Multiple examples of constructing a parabola

Further on, they could apply the *what-if* strategy by varying the initial conditions, e.g. to study the locus of points when the straight line is replaced by a circle – a question whose answer would not be necessarily known to the teacher.

The formation of skills for inquiry based learning (or at least some of its components) reduces the number of actions or tries leading to the formulation of a specific hypothesis.

To have the ability of attacking problems by different means, of looking at them from different angles requires time and efforts of different nature, which should be part of the school activities. It is of vital importance for the pre-service teachers in mathematics and information technologies to have experience and competencies in implementing the inquiry based learning of mathematics, on one hand, and in the inquiry based teaching of mathematics at different levels, on the other. This is a necessary condition for the teachers to cultivate key inquiry competencies in their students.

A crucial aspect of the *constructionism* as an educational philosophy giving rise to the IBL is fostering situations in which the teacher has to join the students as an authentic co-learner. As Seymour Papert expresses it picturesquely in [5], *the best way to become a good carpenter is by participating with a good carpenter in the act of carpentering. By analogy, the way to become a good learner is by practicing with a good learner in an act of learning. In other words, the student should encounter the* ***teacher-as-learner*** *and share the act of learning.* It is in the ideology behind the *VirMathLab* to enable the pre-service teachers to act as learners on their own and to share the act of learning with their pupils.

The examples in Fig. 4 illustrate the idea that the resources could be used for studying a specific topic (e.g. *Functions*) as well as for implementing the knowledge gained in a new situation: a game (http://www.math.bas.bg/omi/cabinet/content/bg/html/d24024.html) and art compositions (http://www.math.bas.bg/omi/cabinet/content/bg/html/d25016.html).

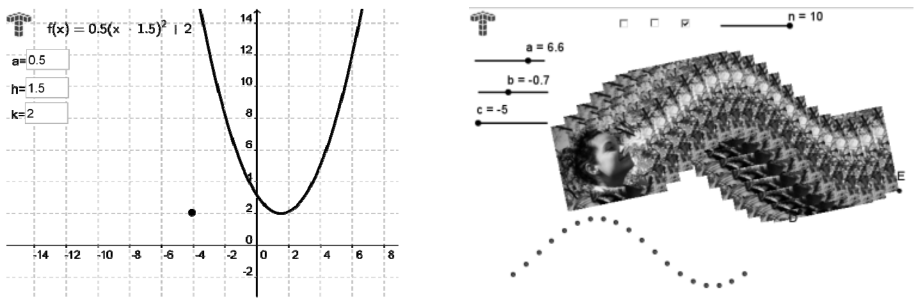

Fig. 4. Examples of the "Games" and "Art" sections

CONNECTION OF THE VIRMATHLAB WITH CURRENT PROJECTS

Another important feature of the *VirMathLab* is that it will be dynamically enriched with resources developed within educational projects with two-way links. For instance, the dynamic *Snowflakes* file (http://www.math.bas.bg/omi/mascil/task-Snowflakes-bg.html) is linked to the scenario "Let us make a snowflake" within the *MaSciL* project which in turn refers to the *Snowflakes* file (Fig. 5, left).

Fig. 5. Examples of links with MaSciL and DynaMat scenarios

Dynamic files related to resources within the *DynaMat* project, e.g. on finding geometric patterns as a game of dynamic explorations [3], are shown in Fig. 5 (right).

Although under development the *VirMathLab* has already been used by teachers, students, parents (even grandparents) from various regions of the country. The first impressions of the users' feedback are very encouraging in terms of the broad spectrum of applicability of the *VirMathLab* at different levels of inquiry based learning. We continue to work with teachers in the frames of the projects *Mascil, KeyCoMath, Scientix2* by organizing and delivering courses and seminars in support of the inquiry based mathematics education by means of digital technologies.

CONCLUSIONS

The education of pre-service teachers in mathematics and in information technologies should include the formation of skills and competencies of using virtual didactic means and knowledge about the available resources. There exist already good practices of pre-service students at the *Faculty of Mathematics and Informatics* (Sofia University) using the *VirMathLab* when working on a project within the university subject *Methods of teaching mathematics,* as well as in lesson preparation.

It is our strong conviction that the resources of the *VirMathLab* could be successfully used for solving pedagogical problems in the context of university courses on *Didactical means, Audio-visual information technologies in education, Computer languages and systems for education, Methods of teaching mathematics,* etc. Although there could be an optional course on working with *GeoGebra* as a main specialized dynamic geometry software behind the dynamic files of the *VirMathLab*, our experience shows that most of the users do not need training on the software itself. In the courses we organize for teachers' professional development the emphasis is on the methods of using the resources, on the experiencing the inquiry based learning at different levels, on gaining the competence of learning to learn and sharing the act of learning, on the new type of communication with the pupils – as co-members of a research team.

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REFERENCES

[1] Banchi, H, Bell, R. *The-Many-Levels-of-Inquiry*, retrieved February 8, 2014 from http://www.miseagrant.umich.edu/lessons/files/2013/05/

[2] Chehlarova, T., Sendova, E. IMBE in the secondary school: Overview and examples in a Bulgarian context, in Baptist, P., Raab, D. (eds.): Implementing Inquiry in Mathematics Education, Bayreuth 2012. ISBN 978-3-00-040752-9

[3] Chehlarova, T., Sendova, E. Finding geometric patterns as a game of dynamic explorations. Scientia iuvenis, Book of Scientific Papers, Scientific guarantee: prof. RNDr. Ľubomír Zelenický, CSc. Slovak republic, Constantine the Philosopher University in Nitra, 2013, pp. 487-494 ISBN 978-80-558-0390-6

[4] Kenderov, P. Sendova, E. *Inquiry Based Mathematics Education (IBME) and Gifted studentsm,* in Baptist, P., Raab, D. (eds.): Implementing Inquiry in Mathematics Education, Bayreuth 2012. ISBN 978-3-00-040752-9

[5] Papert, S. Introduction to *Logo Philosophy and Implementation.* LCS Inc. 1999, ISBN 2-89371-494-3

[6] Sendov, B., Dicheva, D., A Mathematical Laboratory in Logo Style, in Lovis, F. and Tagg E.D. (eds.), Computers in Education – Proceedings of the IFIP TC3 European

Conference on Computers in Education (ECCE'88), Lausanne, Switzerland, North Holland, 1988

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The paper has been reviewed.